Single Variable Production Functions

The production function for a single variable input and output is specified as

$$Q = f\,(X_1 | X_2 \ldots\ldots X_n)$$

Here, X_1 is variable input, $X_2 \ldots . X_n$ are fixed inputs and Q is output.

Average product of X_1 is defined as

$$AP_1 = Q/X_1$$

Due to the operation of the law of diminishing returns, AP_1 must decrease as increases

Marginal product of X_1

$$MP_1 = dQ/dX_1$$

Elasticity of X_1 input

$$E_1 = \frac{dQ}{Q} \bigg/ \frac{dX_1}{X_1}$$

$$= \frac{dQ}{dX_1} \bullet \frac{X_1}{Q} = \frac{dQ}{dX_1} \bigg/ \frac{Q}{X_1} \quad (3.1$$

Elasticity of production $(E_P) = MP_1/AP_1$ (3.1

Let us assume the estimated production function with a single variable input as

$$\hat{Q}_1 = 20 + 150\,X_1 - 1.2\,X_1^2 + e \quad (3.1$$

Equation (3.12) is called quadratic production function or second degree polyno mial function. With single variable input using equations (3.8) (3.9) and (3.11) we ca derive AP_1, and MP_1

$$\hat{Q} = 20 + 150\,X_1 - 1.2\,X_1^2$$

$$AP_1 = Q/X_1$$

$$= 20/X_1 + 150 - 1.2\,X_1$$

$$MP_1 = dQ/dX_1 = 0 + 150 - 2.4\,X_1 \quad (3.13)$$

$$= 150 - 2.4\ X_1$$

$$E_p = \frac{150 - 2.4\ X_1}{20 / X_1 + 150 - 1.2\ X_1} = \frac{MP}{AP_1} \qquad (3.14)$$

Taking derivative of MP_1 (second derivative)

$$\frac{dQ^2}{d^2X_1} = -2.4$$

Since the value is negative, diminishing returns are prevailing. Assuming different values for X_1, we can work out AP_1, MP_1 and E_p and $\hat{Q}$ and these values are given in Table 22.1.

TABLE 22.1 Calculation of AP_1, MP_1 and Q for the Given Production Function.

Assumed values of X_1 (kg)	AP_1(kg)	MP_1 (kg)	E_p	$\hat{Q}$ (Quintal)
1	168.80	147.6	0.87	1.69
10	140.00	126.0	0.90	14.00
20	127.00	102.0	0.80	25.40
30	114.67	78.0	0.68	34.40
40	102.50	54.0	0.53	41.00
50	90.40	30.0	0.33	45.20
60	78.33	6.0	0.07	47.0
70	66.28	-18.0	-0.27	46.40
80	54.25	-42.0	-0.77	43.40

Efficiency of the input use:

This occurs when $MP_1 = P_1/P_q$

Where,

P_1 = Price per unit of input

P_q = Price per unit of output

The details of optimal input and optimal output for corresponding assumed price ratios are furnished in Table 22.2. As the price ratio is declining optimal input levels and optimal output levels are increasing. Please note that the optimal output is almost remaining at the same level, when the price ratio is reduced from 4 to 0.25. In contrast, we notice slight increase in the optimal input level for the corresponding price ratios.

TABLE 22.2 Optimal Level of Input and Output for the Given Production Function.

Price ratio (P_1/P_q)	X_1* (Optimal input) (kg)	Q* (Optimal output) (Quintal)
8	59.17	46.94
7	59.58	46.97
6	60.00	47.00
5	60.41	47.02
4	60.83	47.04
3	61.25	47.06
2	61.67	47.07
1	62.08	47.07
0.5	62.29	47.07
0.25	62.40	47.07

output by less than 1 per cent. Returns to scale can be estimated directly from the b_i values. If $\Sigma b_i > 1$, it implies increasing returns to scale. If it is exactly one, it indicates constant returns to scale and a value less than one indicates decreasing returns to scale. This should be based on test of significance of Σb_i.

Derivation of iso-quant from C-D function

$$Y = A X_1^{b1} X_2^{b2} \ldots\ldots X_n^{bn} \quad (3.23)$$

Setting $Y = Y_0$ in equation (3.23) and solving equation (3.23) for X_1 in terms of X_2, the iso-quant equation is defined as

$$X_1 = \left[\frac{Y_0}{X_2^{b2}\left(AX_3^{b3}\ldots\ldots X_n^{bn}\right)}\right]^{\frac{1}{b1}} \quad (3.24)$$

Where,

Y_0 is a particular level of output defining the iso-quant.

The marginal rate of technical substitution (MRTS) of inputs is defined as the ratio of differentials of the two inputs or as the ratio of their marginal products.

$$MRTS = -\frac{\partial X_1}{\partial X_2} = \frac{\partial Y}{\partial X_2} \Big/ \frac{\partial Y}{\partial X_2} = \frac{b_2}{b_1} \bullet \frac{X_1}{X_2} \quad (3.25)$$

In terms of differential equation (3.25) is specified as

$$\partial Y = \frac{\partial Y}{\partial X_1} \bullet \partial X_1 + \frac{\partial Y}{\partial X_2} \bullet \partial X_2 = 0 \quad (3.26)$$

Advantages of Cobb-Douglas Production Function

1. It is popularly used in agricultural economics research because of its simple functional form which provides for easy computation.
2. It gives theoretically consistent and significant estimates for most of the variables used in the analysis of agricultural data.
3. Elasticities are directly measured.
4. The estimates of this function are mostly consistent with the principle of law of diminishing returns i.e. marginal productivity decreases as the input use increases.
5. Returns to scale are directly estimated.
6. The inverse relationship that exists between marginal rate of substitution and factor proportions is easily computed from Cobb-Douglas function.

Mitscherlich or Spillman Function

$Q = M - AR^{X1}$

Q = Output

M, A and R = Constants

X_1 = Input

***Resistance* function:** It is specified as

$$Q^{-1} = a_0 + \Sigma a_i \, (b_i + X_i)^{-1}$$

***Transcendental* function:** It is specified as

$$Q_i = a \, \pi \, X_i^{bi} \, e^{C_i} \, i$$

CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION (CESPF)

In the Cobb-Douglas production function, if elasticity of substitution (σ) is equal to 1 then, it gives constant returns to scale. Such production function is called Leontief input-output production function.

In the linear production function elasticity of substitution is infinite*. CESPE was formulated by Arrow, Chenery, Minhas and Solow. It takes the following form

$$Y = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho} \tag{3.27}$$

Where,

Y = Output

K = Capital

L = Labour

Here ϒ is called efficiency parameter and it is equal to A in Cobb-Douglas production function.

δ is distribution parameter and its value lies between 0 and 1, $(0 \le \delta \le 1)$ and ρ is substitution parameter and its limits range from –1 to α, $(-1 \le \rho \le \alpha)$.

Elasticity of substitution $(\sigma) = 1/1+\rho$

The value of elasticity of substitution, $(\sigma) = \alpha < \alpha < 0$

If $\rho = 0$ and $\sigma = 1$, CESPF becomes Cobb-Douglas production function

If $\rho = -1$ and $\sigma = \alpha$, CESPF becomes linear production function

If $\rho = \alpha$ and $\sigma = 0$, CESPF takes the form of Leontief input-output production function.

Assuming constant returns to scale in the CES production function, we have marginal products of capital and labour as

$$\frac{\partial Y}{\partial L} = \sigma \gamma^{-\rho}\left(\frac{Y}{K}\right)^{1+\rho} \tag{3.28}$$

---

* $Y = a_1X_1 + a_2X_2$, There elasticity of substitution (σ) in this case is

$$\delta = \frac{d\left(\frac{X_1}{X_2}\right) \Big/ \frac{X_1}{X_2}}{d\left(\frac{a_2}{a_1}\right) \Big/ \frac{a_2}{a_1}}$$ Since a_1, a_2 are both constants, then $d\left(\frac{a_2}{a_1}\right) = 0$. This means denominator becomes zero. Therefore, $\sigma = \text{Infinite } (\infty)$.

$$\frac{\partial Y}{\partial L} = (1-\sigma)\gamma^{-\rho}\left(\frac{Y}{L}\right)^{1+\rho} \quad (3.29)$$

Dividing equation (3.28) by equation (3.29), we get the MRTS

$$MRTS = \frac{\partial L}{\partial K} = \frac{\sigma}{1-\sigma}\left(\frac{L}{K}\right)^{1+\rho} = \frac{P_K}{P_L}$$

Where,

P_K = Price of capital

P_L = Price of labour

CES production function can be estimated directly by using maximum likelihood technique or it can be indirectly estimated by using the relationship between average productivity of labour and its wage rate, which gives the value of elasticity of substitution as the coefficient for the wage rate.

$$\log Y = \frac{1}{1+\rho}\log P_L - \frac{1}{1+\rho}\log\left[\gamma^{-\rho}(1-\delta)\right] \quad (3.30)$$

$$\log P_L = (1+\rho)\log Y - \rho \log Y + \log(1-\delta) \quad (3.31)$$

$$C = \gamma^{-\rho}(1-\delta) \quad (3.32)$$

Where,

C = Constant

Using equations (3.30), (3.31) and (3.32) we get log-linear CESPF,

$\log Y = \sigma \log P_L - \sigma \log C.$

Here CES function implies a log linear relationship between average productivity of labour and wage rate.

Here σ is the value of elasticity of substitution.

$$\sigma = \frac{d\left(\frac{K}{L}\right)}{d\left(\frac{P_L}{P_K}\right)} \bullet \frac{P_L L}{P_K K}$$

Using logarithms CESPF can be estimated as

$$\log Y = \log \gamma - \frac{1}{\rho}\log\left[\delta_1 T^{-\rho} + \delta_2 L^{-\rho} + (1-\delta_1-\delta_2)K^{-\rho}\right]$$

Where,

Y = Output in Rs.

T = Cropped area in ha.

L = Labour in Rs.

K = Interest on fixed capital in Rs.

For application and interpretation of the parameters of CES production function, please refer to Yotopoulos and Nugent (1976)*.

QUADRATIC PRODUCTION FUNCTION

General Form

$$Y = a + bX - cX^2 \tag{3.33}$$

Where

Y = Yield

X = Input say, nitrogen

a = Yield due to fixed factors like phosphorous, potash, seed, irrigation, human labour *etc.*, which have been kept constant.

The simple quadratic equation with a minus sign before c denotes diminishing marginal returns. It allows both a declining and negative marginal productivity, but not both increasing and decreasing marginal products.

Sppose the estimated quadratic equation is

$$Y=1{,}146 + 5.22\ X - 0.003X^2$$

$-0.003\ X^2$ indicates that the curve is sloping downwards, which means that it shows diminishing rate of return. The optimum dose of N and Y (output) can be obtained as presented below:

MPP_x of the total product curve is

$$dY/dX = 5.22 - 0.006X$$

If P_x =Rs. 20/kg

and P_y = Rs. 500/Qtl or Rs. 5/kg

Then,

$$5.22\text{-}0.006\ X = P_x/P_y = 20/5 = 4$$

$$= 5.22\text{-}0.006\ X = 4$$

$$= -0.006\ X = 4\text{-}5.22$$

$$= -0.006\ X = -1.22$$

$$X = 203.33 \text{ kg}$$

Optimum dose of nitrogen is 203.33 kg. Now substituting this value of X in the original equation

$$Y = 1{,}146 + 5.22\ (203.33) - 0.003(203.33)\ (203.33)$$

$$= 1{,}146 + 1{,}061.38 - 124.03$$

$$= 2{,}207.38 - 124.03$$

$$= 2{,}083.08 \text{ kg}$$

This is the output at the most profitable level of nitrogen application. Suppose we want to estimate the response of nitrogen, substitute the value of X in the equation

$$Y = bX\text{-}cX^2$$

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* Yotopoulos P.A. and Nugent, J.B. Economics of Development. Empirical Investigation. Harper & Row, Pub. New York, 1976, pp. 47-70.

Then,

$Y = 5.22\ (203.33) - 0.003\ (203.33)^2$

$= 1{,}061.38 - 124.03$

$= 937.35$ kg

Therefore for 203.33 kg of nitrogen, the yield is 937.35 kg

CONSTRAINED OUTPUT MAXIMIZATION–ALGEBRAIC DERIVATION

Let us assume that the Cobb-Douglas production function is in the following form

$Q = BoL^a K^b$

Bo = It is an intercept term and is called efficiency parameter, it measures the effect of technology on the output of crop, (Q)

L = Units of labour used in producing a crop

K = Units of capital used in producing a crop.

Marginal product of labour (MP_L) is derived as

$$MP_L = \frac{\partial Q}{\partial L} = aBoL^{a-1} K^b$$

$$= a\left(BoL^a K^b\right)L^{-1}$$

Substitute $BoL^a K^b = Q$

$$= a\frac{Q}{L}$$

$$= a(AP_L)$$

Where,

AP_L is average product of labour. The marginal product of capital (MP_K) is defined as

$$MP_K = \frac{\partial Q}{\partial K} = bBoL^a K^{b-1}$$

$$= b(BoL^a K^b)\ K^{-1}$$

$$= b\frac{Q}{K}$$

$$MP_K = b\ AP_K$$

Where, AP_K = average product of capital input.

$$\text{MRTS L, K} = \frac{\partial Q / \partial L}{\partial Q / \partial K} = \frac{a}{b} \bullet \frac{K}{L}$$

To determine the equilibrium conditions, we have to maximize the output subject to the cost constraints.

Maximize $\hat{Q} = f(L, K)$ (3.34)

Subject to $M^\circ = wL + rk$ (3.35)

Where,

M^o = Money available in specified units, such as 1 unit= Rs. 100 or Rs. 1000, *etc.*

r = Price (interest rate) per unit of capital

W = Wage rate per unit of labour

We can solve the above problem using Lagrangean multiplier method. Here the constrained equation is rewritten as

$$M^\circ = wL + rk = 0 \tag{3.36}$$

Multiply the constrained equations (3.36) by constant term, λ, which is called Lagrangean multiplier.

$$\lambda(M^\circ = wL + rK) = 0 \tag{3.37}$$

If there is one constrained equation we should use one λ and if there are two constraints, we should use two λs. Here Lagrangean multiplers are undefined constants which are used for solving constrained equations.

Now let us write Lagrangean function as

$$V = \hat{Q} + \lambda(M^\circ = wL + rK) \tag{3.38}$$

Maximization of the equation (3.38) implies maximization of output subject to cost constraint. To achieve this we have to take partial derivatives of equation (3.38) with respect to L, K and λ and these should be made equal to zero and solve for L, K and λ.

$$\frac{\partial V}{\partial L} = \frac{\partial Q}{\partial L} + \lambda(-w) = 0 \tag{3.39}$$

Then,

$$\frac{\partial V}{\partial L} = \frac{\partial Q}{\partial L} + \lambda(-r) = 0 \tag{3.40}$$

Similarly,

$$\frac{\partial V}{\partial \lambda} = M^\circ - wL - rK = 0 \tag{3.41}$$

Solving equation (3.39) and (3.40) for λ we get

$$\frac{\partial Q}{\partial L} = \lambda w \tag{3.42}$$

$$\lambda = \frac{\partial Q}{\partial L} \Big/ w = \frac{MP_L}{w} \tag{3.43}$$

(or)

$$\lambda = \frac{\partial Q}{\partial L} \Big/ r = \frac{MP_K}{r} \tag{3.44}$$

These two equations must be identical

$$i.e., \frac{MP_L}{MP_K} = \frac{w}{r}$$

Equation (3.44) and (3.45) are first order necessary conditions. The agricultural business firm is said to be in equilibrium where it equates marginal productivities of factors to ratio of their prices.

Equation (3.44) now can be written as

$$\frac{MP_L}{w} = \frac{MP_K}{r} = \lambda$$

The second order sufficient condition should be satisfied for profit maximization under constrained cost equation. These conditions are specified as

$$\frac{\partial^2 Q}{\partial L^2} < 0 \text{ and } \frac{\partial^2 Q}{\partial K^2} < 0$$

$$\left(\frac{\partial^2 Q}{\partial L^2}\right)\left(\frac{\partial^2 Q}{\partial K^2}\right) > \left(\frac{\partial^2 Q}{\partial L\, \partial K}\right)^2$$

Then conditions are implied in the convex property of iso-quant*.

Constrained Output Maximization–*Numerical example*

Consider an estimated Cobb-Douglas production function, which is given as

$$\hat{Q} = 80L^{0.43} K^{0.54}$$

with $\bar{Q} = 1{,}800$ units of output of crop (3.47)

Let us assume that wages per labour unit = Rs. 30 = w
and price per unit of capital = Rs. 40 = r

$$\frac{\partial Q}{\partial L} = 34.4\, K^{0.54} / L^{0.57}$$

$$\frac{\partial Q}{\partial K} = 43.2\, L^{0.43} / K^{0.46} \quad (3.48)$$

For output maximization, we should have (3.49)

$$\frac{\partial Q / \partial L}{\partial Q / \partial K} = \frac{w}{r}$$

$$= \frac{34.4K^{0.54}/L^{0.57}}{43.2L^{0.43}/K^{0.46}} = \frac{30}{40} \quad (3.50)$$

0.54 and 0.43 are the elasticity coefficients i.e., (a and b) of capital and labour respectively.

* Henderson and Quandt, Micro Economic theory, McGraw-Hill, 1958, pp. 49-54.

$= 40\,(34.4\,K^{0.54} / L^{0.57}) = 30(43.2\,L^{0.43} / K^{0.46})$

$= 1{,}376\,K^{0.54+0.46} \qquad = 1{,}296\,L^{0.43+0.57}$

$= 1{,}367\,K \qquad = 1{,}296\,L$

$K = 0.94\,L$ (3.51)

$L = 1.06\,K$ (3.52)

Equation (3.51) and (3.52) are expansion paths. To solve for optimal value of L, substitute K = 0.94 L in equation (3.47)

$Q = 80\,L^{0.43}\,(0.94\,L)^{0.54}$

$Q = 75.2\,L^{0.43\,+\,0.54}$

$Q = 75.2\,L^{0.97}$ (3.53)

Substitute Q = 1,800 in equation (3.53)

$1{,}800 = 75.2\,L^{0.97}$

$$L^{0.97} = \frac{1{,}800}{75.2} = 23.94$$

$$L^{*} = 23.94^{\frac{1}{0.97}}$$

$$L^{*} = 26.41$$

Substitute L^{*} = 26.41 and Q = 1,800 (assumed level) in equation (3.47)

$1{,}800 = 80\,(26.41)^{0.43}\,K^{0.54}$

$1{,}800 = 326.93\,K^{0.54}$

$K^{0.54} = 5.51$

$$K^{*} = (5.51)^{\frac{1}{0.54}} = 5.51^{(1.85)}$$

$$K^{*} = 23.58$$

To find out the value of λ, substitute value of Q = 1,800, a = 0.43 and b = 0.54 and L^{*} =26.41 and K^{*} = 23.58 in equations (3.54) and (3.55)

$$\frac{\partial Q}{\partial L}\Big/ w = \lambda \qquad (3.54)$$

$$\frac{\partial Q}{\partial K}\Big/ r = \lambda \qquad (3.55)$$

$$= \frac{0.43\left(\dfrac{1800}{26.41}\right)}{30} = \lambda$$

$$= 0.43\left(\frac{1800}{26.41}\right) = 30\lambda$$

$$\lambda = \frac{29.31}{30} = 0.98$$

$$\frac{\partial Q}{\partial K}\Big/ r = \lambda$$

$$\frac{\partial Q}{\partial K} = r$$

$$0.54\left(\frac{1800}{23.58}\right) = 40\lambda$$

$$= 41.22 = 40\lambda$$

$$\lambda = \frac{41.22}{40} = 1.03$$

Difference in λ values here is due to fractional errors. Here λ is defined as marginal product of money. If one rupee is invested on the production of output the business firm would get one rupee and 3 paise return. So, it is the contribution to the output.

$\frac{MP_L}{w} = \frac{MP_K}{r} = \lambda$, this condition is also satisfied in the above numerical example.

CONSTRAINED COST MINIMIZATION-ALGEBRAIC EXPRESSION

This is also called dual of output maximization. Here we minimize the costs subject to production function. This problem is in particular specified so as

Minimize $M^o = wL + rK$ (3.56)

Subject to $Q^0 = f(L, K)$ (3.57)

Equation (3.56) is a cost equation and equation (3.57) is an iso-quant equation.

The Lagrangean function (Z), is now specified with Lagrangean multiplier, μ.

$Z = wL + rK + \mu\,[(Q^0 - f(L, K)]$ (3.58)

Taking partial derivatives of equation (3.58) with respect to L, K and μ and equating them to zero we can minimize the cost.

$$\frac{\partial Z}{\partial L} = w - \mu\left(\frac{\partial Q^0}{\partial K}\right) = 0 \quad (3.59)$$

$$\frac{\partial Z}{\partial K} = r - \mu\left(\frac{\partial Q^0}{\partial K}\right) = 0 \quad (3.60)$$

$$\frac{\partial Z}{\partial \mu} = Q^0 - f(L, K) = 0 \quad (3.61)$$

Equation (3.59), (3.60) and (3.61) are first order necessary conditions for cost minimization. Second order sufficient condition is specified as

$$\frac{\partial^2 Q}{\partial L^2} > 0 \text{ and } \frac{\partial^2 Q}{\partial K^2} > 0 \quad (3.62)$$

and

$$\left(\frac{\partial^2 Q}{\partial L^2}\right)\left(\frac{\partial^2 Q}{\partial K^2}\right) < \left(\frac{\partial^2 Q}{\partial L\,\partial K}\right)^2 \qquad (3.63)$$

MEASURING EFFICIENCY IN AGRICULTURAL PRODUCTION – APPLICATION OF FRONTIER PRODUCTION FUNCTION

Efficiency is very much required to achieve the desired growth in agricultural production. Economic efficiency by definition is the sum of technical (production) efficiency and allocative (price) efficiency under different levels of technology. The existing technology is represented by the choice of production function. For example, the particular type of production function, which is selected for the data in question represents technology *i.e.*, technology is represented by type of production function. The concepts of efficiency measures in agricultural production have far reaching implications for policy measures *viz.*, price policy, input, income distribution, land ceilings, *etc.* Yotopoulos and Lau (1971); Huang *et al.*, (1986); Rana (1982) *etc.*, measured the efficiency measures in agricultural production. The theory of profit function approach was developed and applied to agriculture by Lau and Yotopoulos (1973). Let us assume that profit function in logarithmic form be given as

$$\ln \pi_i = \ln f(L_{1i}, Y_{1i}, Y_{2i}, Y_{3i}, Y_{4i}, Y_{5i}, Y_{6i}) + \ln E_i$$

Where,

π_i = Gross profit (gross income–cost of laobur i.e., family and hired labour) of i^{th} farm (i=1 to n) from the production of crop.

L_{1i} = Man equivalent human labour employed by i^{th} farm in the production of crop.

Y_{1i} = Land area in ha under the crop cultivated by i^{th} farm.

Y_{2i} = Costs in rupees on chemical fertilizers incurred by i^{th} farm.

Y_{3i} = Costs of farmyard manure in Rs. Incurred by i^{th} farm.

Y_{4i} = Costs of seeds incurred by i^{th} farm in Rs.

Y_{5i} = Costs on plant protection incurred by the i^{th} farm in Rs.

Y_{6i} = Miscellaneous costs incurred by the i^{th} farm which include depreciation, interest on working costs, interest on fixed capital, cost of bullock labour, land revenue and machine labour incurred by the i^{th} farm.

All these variables are measured in varying units by the i^{th} farm at a particular point of time over space *i.e.*, (during a particular season over an area).

Here the term π is net of human labour cost only. The other variables *viz.*, fertilizers, manures, seeds and plant protection chemicals are placed within the category of fixed inputs. This is because allocation decisions of these inputs thus have little bearing on the profit maximizing behaviour of the farm. The production function is specified as

$$\hat{\pi} = KL^{Bo} \cdot \underset{j=1}{\overset{6}{\pi}} Y_{ij}^{Bj}$$

Cobb-Douglas production function is thus derived as follows:

Now the profit function is expanded as

$$\pi_i = KL^{Bo} Y_1^{B1} Y_2^{B2} Y_3^{B3} Y_4^{B4} Y_5^{B5} Y_6^{B6} e^{u} \qquad (3.64)$$

Let the estimated profit function be written as

$$\hat{\pi}_i = 12.94\, L^{-0.224}\, Y_1^{0.3241}\, Y_2^{0.1981}\, Y_3^{0.1624}\, Y_4^{0.1506}\, Y_5^{0.0896}\, Y_6^{0.2801}$$

$$\sum b_i = 0.981 \qquad (3.65)$$

Since $\sum b_i = 0.981$, diminshing returns to sale are in operation on the farms. Let us further assume that all the variables are significant at 0.05% and 0.1% level of probability.

$R^2 = 0.87$

$SSE = 0.823 = \sigma^2$

$\sigma = 0.907$

Let the Cobb-Douglas production function be specified as

$\dot{Q} = AL^{\alpha 0}\, Y_1^{\alpha 1}\, Y_2^{\alpha 2}\, Y_3^{\alpha 3}\, Y_4^{\alpha 4}\, Y_5^{\alpha 5}\, Y_6^{\alpha 6}\, e^{U}$. Its parameters are astimated as (3.66)

$$A = \frac{1}{K^{(1-Bo)}},\quad \alpha_0 = \frac{-Bo}{(1-Bo)},\quad \alpha_1 = \frac{B_1}{(1-Bo)},\quad \alpha_2 = \frac{B_2}{(1-Bo)} \qquad (3.67)$$

$$\alpha_3 = \frac{B_3}{K^{(1-Bo)}},\quad \alpha_4 = \frac{-B_4}{(1-Bo)},\quad \alpha_5 = \frac{B_5}{(1-Bo)},\quad \alpha_6 = \frac{B_6}{(1-Bo)}$$

Using the above relationships of (3.67) we can derive coefficients of equation (3.66).

$$Q_i = 0.0436\, L^{0.183}\, Y_1^{0.2648}\, Y_2^{0.2618}\, Y_3^{0.1327}\, Y_4^{0.123}\, Y_5^{0.0782}\, Y_6^{0.2298} \qquad (3.68)$$

Based on mean levels, the optimal demand for labour is worked out as 208.24 mandays. Here, the estimated profit function is in Cobb-Douglas form. The geometric means of Y_1 to Y_6 and Q are considered and optimal demand for labour is worked out.

$$\text{Efficiency index of labour} = \frac{\text{Actual labour used on } i^{th} \text{ farm}}{\text{Optimal demand of labour}}$$

estimated from Cobb-Douglas function

$$= \frac{103.36}{208.24}$$

$$= 0.4964$$

Here, the labour use efficiency is nearly 50 per cent. Similarly optimal values for the remaining variables can be worked out based on elasticity coefficients of the variables and their geometric means.